

# Nonlinear dynamics of multiwave mixing in optical microcavities

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## Abstract:

The generation of optical Kerr frequency combs by microresonators has attracted much interest in the last few years [1]. The equidistant and highly resolved spectral lines of these combs are expected to help facilitate numerous applications such as optical clocks, sensing and spectroscopy. To date, the theoretical descriptions of microresonator frequency combs was mostly carried out using a modal expansion approach, which describes the slow evolution of the comb spectrum using time-domain rate equations [2]. An alternative description of microresonator frequency combs was recently proposed [3] that allows for a time-domain description of the comb as mode-locking of soliton solutions of the driven and damped nonlinear Schrödinger equation (NLSE), which was previously applied for the description of CW-driven fiber-loop soliton memories [4-5].

In this presentation, we apply the driven and damped NLSE model to demonstrate that comb generation can be given a simple interpretation in terms of modulational instability (MI) of the CW pump field [4]. We present a theoretical stability analysis applicable to microresonators combs, which provides analytical expressions for the conditions under which stable comb generation may occur. Although MI of the CW solutions may also occur in the normal dispersion regime [4], we show that the generated combs tend to have a finite lifetime in this case. On the other hand, we predict that stable comb generation and trains of mode-locked dark soliton pulses may be generated in optical microresonators by using a dual-frequency input pump [6]. Important qualitative insights into the nonlinear dynamics of the comb generation in both normal and anomalous cavity dispersion regimes can be obtained by using truncated three or four wave models. The results of finite mode truncations are compared and found to be in good agreement with the full numerical simulations.

## References:

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